Nothing is better than something – Perspective of Structural Optimization in Civil Engineering Applications

S. Lowhikan and H.M.S.T. Herath

Abstract

Structural optimization of solids aims to find the optimal designs of structures by minimizing a constrained objective function such as the material compliance within a given problem domain. This constrained optimization problem is subjected to a set of displacement and load boundary conditions which in turn will be minimized with respect to a structural parameter. Although various structural optimization techniques have a sound mathematical basis, the practical constructability of optimal designs poses a great challenge in the manufacturing stage. The recent development in additive manufacturing partially side-steps this problem predominantly in the domain of Mechanical Engineering. However, in Civil Engineering structures, there is a great possibility of utilizing these optimization tools, especially in precast constructions. Currently, there is only a limited number of unified frameworks which output ready to manufacture parametric Computer-Aided Design (CAD) of the optimal designs. From a generative design perspective, it is essential to have a single platform that outputs a structurally optimized CAD model because CAD models are an integral part of most industrial product development and manufacturing stages. This study focuses on developing a novel unified workflow handling both topology and size optimization in a single parametric platform (Rhino-Grasshopper) which outputs a ready-to-manufacture CAD model with the assessment of their structural integrity. In the proposed method, the first topology optimized pixel model is generated for any two-dimensional problem and converted into a one-pixel-wide chain model using skeletonization. From the obtained skeleton, a spatial frame structure is extracted, and its members are sized optimally. Finally, the CAD model is generated using Constructive Solid Geometry (CSG) trees and its structural performance is assessed. In addition, industry-standard structural sections can be assigned to the CAD model to be analysed and designed in accordance with standard codes of practice.

1. Introduction

Structural optimization is a mathematical methodology for finding the optimal design of structures with minimal stress, weight or compliance for a given amount of material and constraints. It improves structural designs by increasing stiffness, lowering material consumption, and reducing production time [1]–[3]. This will lead to low-carbon developments with efficient material usage.

Structural optimization techniques can be broadly categorized into three categories such as topology optimization, size optimization and shape optimization as shown in Figure 1. In topology optimization, the material distribution is optimized for a given set of loadings and boundary conditions within a defined design domain. Size optimization is a process of optimizing the cross-sections by increasing the stiffness whereas in shape optimization the optimizing variable is nodal coordinates [4].

Even though various studies have been done using these structural optimization techniques and produced efficient designs, the implementation of those designs in the construction industry is very limited due to the complex geometries of the optimized design outputs [5]. The current advancement in additive manufacturing partially avoids this problem predominantly in the field of Mechanical Engineering. However, Civil Engineering applications are sparse, and there is a lot of potentials for these techniques to be used, especially in precast constructions.

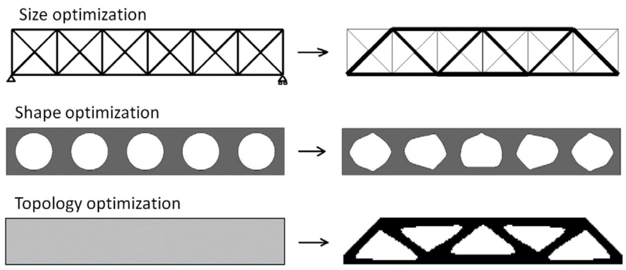

Figure 1: Comparison of topology, size, and shape optimization based on a structural mechanics problem [4].

In addition, the additive manufacturing techniques are very expensive in the local context and if the structural engineers want to use the power of optimization in the Civil industry, they need to produce designs that can be manufacturable or fabricable using the available standard sections.

However, there is only a limited number of unified frameworks which output ready to manufacture parametric Computer-Aided Design (CAD) of the optimal designs. CAD models are an integral part of most industrial product development and manufacturing [5], [6]. Therefore, it is essential to have a single platform that outputs structurally optimized CAD models which can be locally manufacturable using available resources.

This paper presents a novel approach to produce ready to manufacture CAD models of structurally optimized

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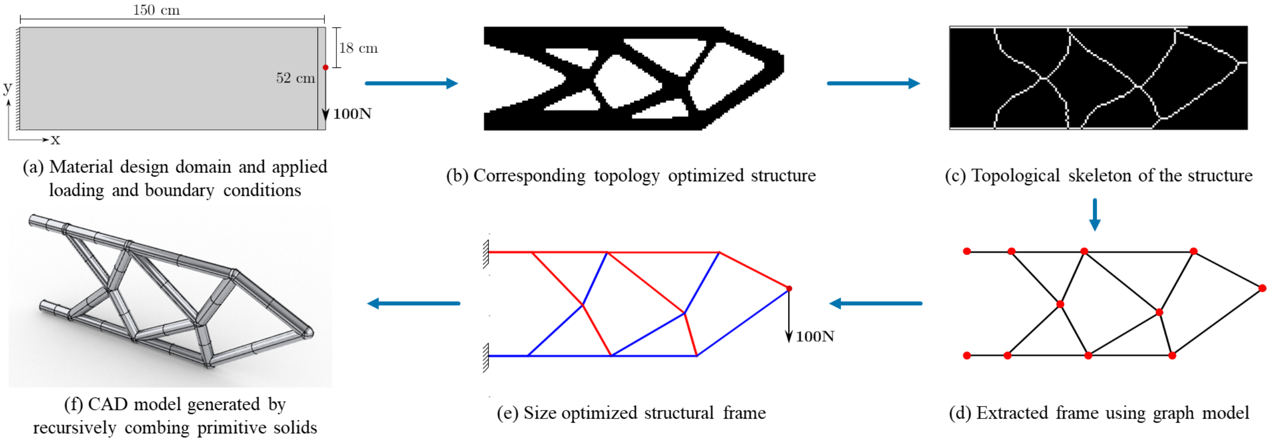

Figure 2: CAD model generation workflow from the topology optimized structure for a cantilevered beam example

designs. This section gives a brief overview of this research project by highlighting the background, research gap, and motivation for the study. Section two describes the methodology adopted to obtain the optimized CAD models and each subsection briefly describes each step. Section three discusses the results of a few examples using the suggested method. Section four concludes the paper by outlining the outcomes of the study.

2. Methodology

In the proposed method, first, any two-dimensional surface is discretized into a number of finite elements (equivalent to finite size pixels in 2D) and a topology optimized 2D binary image is generated. Then, the obtained pixel model is converted into a one-pixel-wide chain model using a skeletonization algorithm. From the obtained skeleton, a spatial frame structure is extracted and represented by a graph model. Thereafter sizes of the frame members are optimized. Finally, the CAD model is generated using constructive solid geometry trees and its structural performance is assessed. In addition, industry-standard structural sections can be assigned to the model to be analysed and designed under standard codes of practice.

Figure 2 illustrates this procedure briefly using a cantilever example. The proposed workflow is implemented in Rhinoceros CAD software utilizing the visual programming tool Grasshopper in a Python coding environment.

2.1 Topology Optimization

Topology optimization is focused on finding the optimal material distribution within a specified design domain. In this workflow standard density-based topology optimization using the modified Simplified Isotropic Material with Penalization (SIMP) method is utilized [7].

The topology optimization problem of a finite element discretized solid is given by [5], [8], [9],

$$\text{minimize} \quad C_{(\rho)} = f u_{(\rho)}$$

$$\text{subject to} \quad K_{(\rho)} u = f$$

$$\text{design constraint:} \quad \frac{V_{(\rho)}}{\bar{V}} \leq V_f$$

$$\text{design variable:} \quad 0 \leq \rho \leq 1$$

where $C_{(\rho)}$ is the objective function, ρ is the vector of relative element densities, u is the displacement vector, $K_{(\rho)}$ is the global stiffness matrix, f is the global external force vector, $V_{(\rho)}$is the material volume, $\bar{V}$ is the design volume and V_f is the prescribed volume fraction. The relative density of each element is constrained to be $0 \leq \rho_i \leq 1$ with $i \, \epsilon \, \{1,2,3 \ldots n_e\}$ where n_e is the number of elements in the domain.

The global stiffness matrix K and vector f are assembled from the n_e element contributions K_i and f_i respectively. In this paper, we discretize the design domain with a structured grid and hexahedral linear elements. In each element, the material is isotropic and homogeneous, and Young's modulus E is penalized depending on the relative density ρ_i according to,

$$E_{(\rho_i)} = E_{min} + \rho_i^p (\bar{E} - E_{min})$$

where $\bar{E}$ is the prescribed Young's modulus of the solid material, ρ and E_{min} are two algorithmic parameters. The penalization parameter $\rho \leq 3$ ensures that elements with densities close to $\rho_i = 0$ (void) and $\rho_i = 1$ (solid) are favoured. The small Young's modulus $E_{min} \cong 10^{-9}$of the void material prevents ill-conditioning of the global stiffness matrix when $\rho_i = 0$. Each element stiffness matrix K_i is computed using the corresponding Young's modulus $E_{(\rho_i)}$ with the relative density ρ_i which is constant within an element.

2.2 Skeletonization

Skeletonization is the process of reducing binary objects to 1-pixel wide representations with the same connectivity as the original structure. It can be used to represent a structure's topological skeleton. The 2D binary image data obtained from the topology optimization is sent through the skeletonization process to extract the medial axis of the structure [10], [11].

An object's medial axis is the set of all points that have more than one nearest point on the object's edge. By using

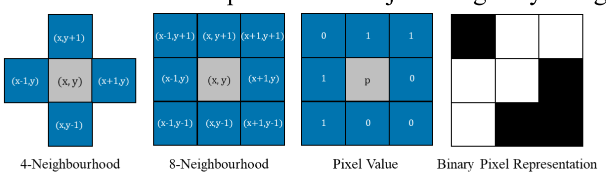

Figure 3: 8-Neighborhoods of a pixel

the 8-neighbourhood properties of a pixel, each pixel is identified and categorized as an edge pixel, end pixel, interior pixel and corner pixel. Figure 3 shows the 8-neighbours of a pixel (x, y) and its coordinates. The 2D skeletonization algorithm proceeds by iteratively removing edge pixels that do not change the topology of the object. This continues until no more pixels can be removed. The pixels erased must satisfy the following three criteria:

1. No end pixel is deleted
2. No connectedness is violated
3. No excessive erosion occurs

Most of the proposed parallel thinning algorithms differ only in the way that they conduct the test to meet these criteria [12]. The modified version of most widely used Zhang and Suen [10] thinning algorithm is used here to extract the medial axis of the structure.

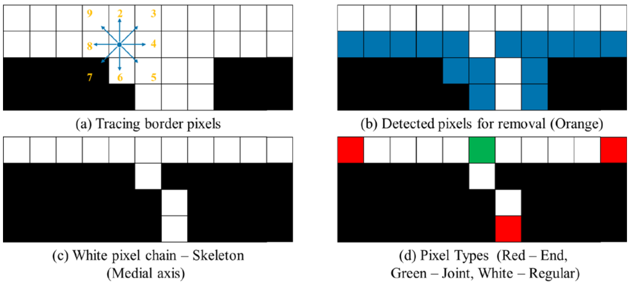

Figure 4: Illustration of pixels categorization

2.3 Frame Extraction

This subsection explains the conversion of topology preserved pixel chains to a simplified 2D frame model using graph algorithms. Characterizing the pixel chain using a graph model which consists of nodes and edges, can give simpler geometric representations. Hence, the skeleton is converted as an unweighted undirected graph to make use of its properties [13].

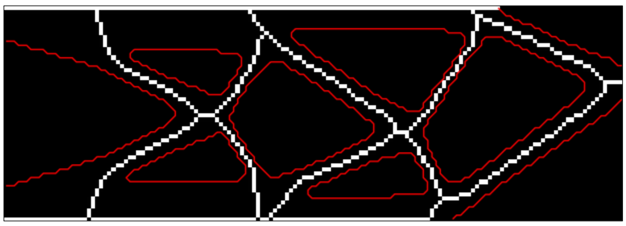

Figure 5: Medial axis of the topology optimized structure

The edges of the graph cannot be converted directly into frame members since it is neither feasible nor practical. If the edges are directly converted into frames, it will lead to short beams in the structure which are ineffective and more complex to further proceed. Therefore, a more compact graph is needed first in order to convert it into the frame model. A compact graph is made by extracting edges which are connecting the important pixels only.

To identify these important pixels, all the pixels need to be categorized first [14]. Topologically, the pixels on a skeleton are classified into three types of pixels (Figure 4),

1. A regular pixel has two 8-neighbored pixels
2. An end pixel has only one 8-neighbored pixel
3. A joint pixel has more than two 8-neighbored pixels

In addition to this, the user needs to tag which pixels to be preserved during any of the conversion processes. Generally, tags are used to represent the locations of loadings and boundary conditions in structural engineering problems. But if the user wanted to keep any pixel for a particular reason, those pixels can be also tagged and preserved. These pixels can be carried out as tagged nodes in the graph model as well.

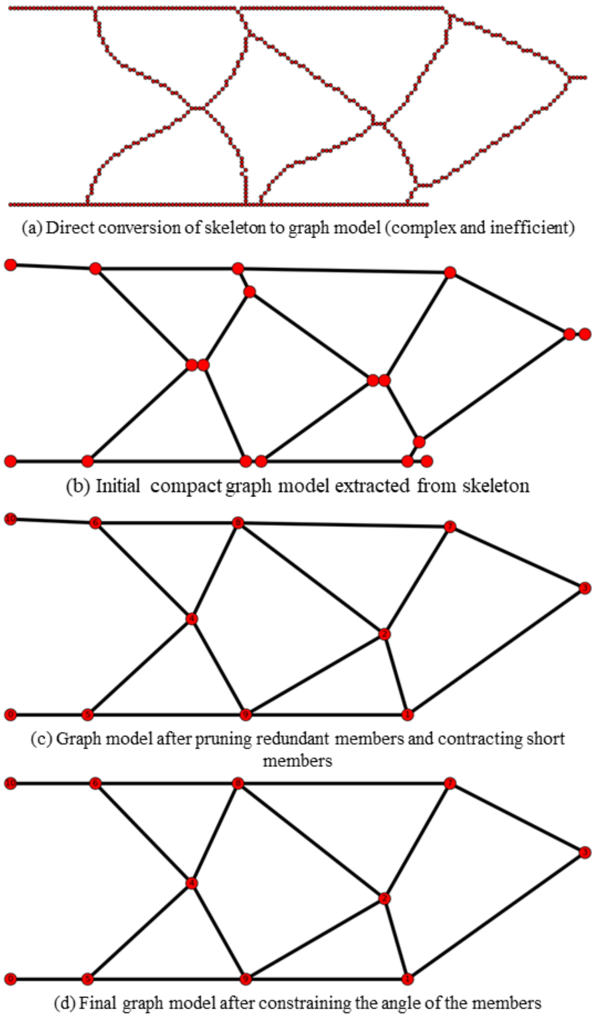

Figure 6: Skeleton-to-frame model generation

End pixel, joint pixel and tagged pixels are important pixels in order to get the edges which gives the basic compact layout of the graph. Here, two pixels in important pixels which are connected through a path consisting of untagged regular pixels are extracted as edge. After the classification of all the pixels, edges are determined by starting from the joint pixel and marching along the 8-neighbours until an important pixel is reached [13]. A graph model made with these edges and nodes will produce a compact graph needed for the further process.

The obtained compact graph model is still inefficient due to the short branches for any structural applications as shown in Figure 6(b). Hence, this is further modified by pruning the redundant branches and contracting the short edges based on a user-defined merge ratio [5].

Skeletons obtained using medial axis transform can produce spurious branches. It is kept even in the compact graph model as redundant edges. One can preprocess the pixel chain after skeletonization to clean the noise branches. Since this paper deals with the case where the useful structural frame members in load transfer are to be identified using a graph model, instead of editing the pixel chain, corresponding redundant edges in the graph model can be pruned. Edges which has a degree of one and which are not connected to any of the tagged nodes in the graph model can be easily identified and pruned until no redundant edge branches are left.

Pruning will not solve the problems of short edges since those edges are not truly redundant as they preserve the topology. Therefore, a user has to define a merge ratio to classify which edges have to be considered as short edges and contracted. A short edge is one that is shorter than the merge ratio of the total length of all members sharing the same node. Two end nodes of the short edges are contracted until no short edge remains in the graph model.

Graph model after the pruning and edge contraction can give a good model to be used for structural analysis applications. One can do a layout optimization in later steps to obtain more optimized nodal positions of the frame model. By doing layout optimization, bending stresses in the members can be further optimized [6], [15]. In this paper, the authors are modifying the nodal positions only by constraining the small angled frames to get a more desirable frame model which is efficient for structural applications. Hence doing this top and bottom frames can be horizontally aligned and gives a better frame model as shown in Figure 6(d).

Although, it is possible to obtain element cross-sections from the pixel chain widths, in this study cross-sections are found using size optimizations of the frames. It would be relatively easy rather than keeping the burden of all the unnecessary chain widths for sizes, which are going to be optimized and changed later.

2.4 Size Optimization of Frame Members

In size optimization, the optimization variable can be a cross-section area or any other parameters describing the cross-section [3]. Size optimization has the same structure as the topology optimization problem in section 2.1, namely,

$$\text{minimize } C_{(A)} = f u_{(A)}$$

$$\text{subject to } K_{(A)}\mathrm{u} = \mathrm{f}$$

$$\text{design constraint: } \frac{V_{(A)}}{\bar{V}} \leq V_f$$

$$\text{design variable: } A_l \leq A \leq A_u$$

The cross-section area considered here is denoted as $A = (A_1, A_2, A_3 \dots\dots A_{n_{elm}})$, where a component A_i represent the area of member i and n_{elm} represent the number of members in the frame model. The lower bound and upper bound of the cross-sectional area, which constrains the member sizes to fit the manufacturing requirements is represented as A_l and A_u in order.

The size optimization of the frames can be effectively done using gradient-based optimization algorithms. The derivatives of the objective function and constraints can be computed easily for the frame members as follows,

derivative of the objective function,

$$\frac{\partial C_{(A)}}{\partial A_i} = -u^T \frac{\partial K_{(A)}}{\partial A_i} u = -\sum_{i=1}^{n_{elm}} u^T \frac{\partial K_{i(A,s)}}{\partial A_i} u$$

derivative of volume constraint,

$$\frac{\partial V_{(A)}}{\partial A} = L_i$$

For structural optimization problems, gradient-based optimization algorithms are commonly used. There are three major gradient-based algorithms such as Sequential Quadratic Programming (SQP) Method of Moving Asymptotes (MMA) and the Optimality Criteria (OC) method [16]. Here, the OC method is utilized for topology optimization because of its numerical efficiency and simplicity compared to other methods in topology optimization problems. For size optimization, any of the gradient-based methods can be chosen, and here MMA method is used.

2.5 CAD Model Generation

The graph model with nodes, edges and optimized member cross-sections is sufficient to create a structural frame model. From the structural frame model details, the CAD model can be easily generated. There are several methods to perform this generation [6], [17], and in this paper, the authors generated solid models using the Constructive Solid Geometry tree technique.

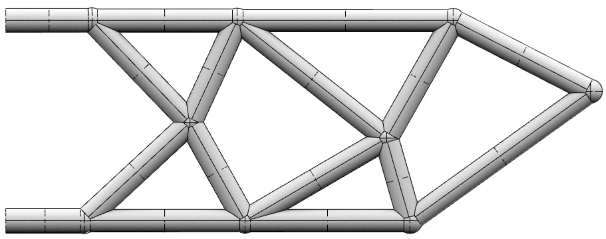

Figure 7: Generated CAD model using CSG tree

First, each member is created by extruding a selected cross-section along the length of the line element, with the area determined from size optimization. After this, a single solid model can be created using Boolean operations. In the case of additive manufacturing products, at each joint, a sphere is created with a maximum radius of all cylinders connected to the joint.

2.6 Assign Section

It is desirable if the engineers can use various available industry-standard sections to manufacture the optimized structure. Since the optimum cross-sectional areas of members have been already found using size optimization, we can make use of that those results to filter the desired shape's dimensions. However, when the cross-sectional shape changes, the slenderness ratio of the member can vary significantly. So to avoid buckling effects, it is necessary to check the buckling of the members at the design stage. The authors are presently working on developing design verification methods for possible failure cases with various sections to improve the workflow.

3. Results and Discussion

This section provides the numerical results of the proposed approach for a selected example. A relatively simple example, which is a cantilever plate is chosen here to discuss the results.

2.1 Cantilever Plate

As shown in Figure 1, a 150 cm × 52 cm plate is selected and one face of the domain is fixed. A point load with a value of F = 100 N is applied at $\bar{y}$ = 18 cm from the top of the plate on the opposite free end. The thickness of the plate is taken as 4 cm for size optimization. Table 1 gives the material properties used for optimizations.

Table 1: Material properties used for the examples

Material property	Value
Solid Young's modulus ($\bar{E}$)	$2.1 \times 10^5\ N/mm^2$
Void Young's modulus (E_{min})	$1 \times 10^{-9}\ N/mm^2$
Poisson ratio (ν)	0.3

In topology optimization, the domain was discretized into 150 × 52 × 1 linear hexahedral elements (3D topology optimization algorithm used considering the future developments) and the objective function $C_{(\rho)}$ is minimized. From the results, the 2D surface results were obtained for the skeletonization. Table 2 gives the parameters used for the topology optimization and Figure 8 shows the convergence of the objective function. Following optimization, the penalization power can be set to $p = 1$ to transform the objective function into structural compliance to be compared with frame structure compliance if needed.

Table 2: Topology optimization parameter

Parameter	Value
Volume fraction ($\bar{V_f}$)	0.5
Penalization power (ρ)	3
Filter radius (r_{min})	1.2
Maximum iterations	200

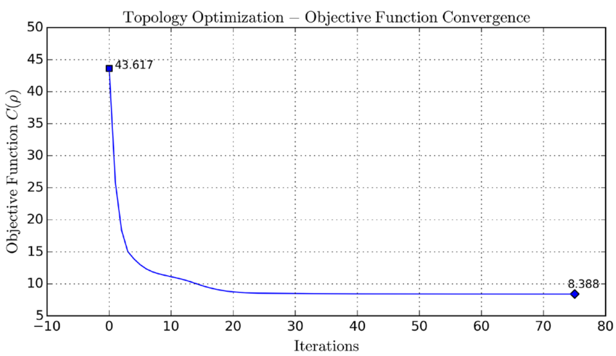

Figure 8: Objective function convergence during topology optimization

Table 3: Size optimization parameters

Parameter	Value
Volume constraint ($V_{(A)}$)	$\leq 15600\ cm^3$
Tolerance (ρ)	10^{-4}
Minimum Area (A_{min})	$3.15\ cm^2$
Maximum Area (A_{max})	$50.25\ cm^2$

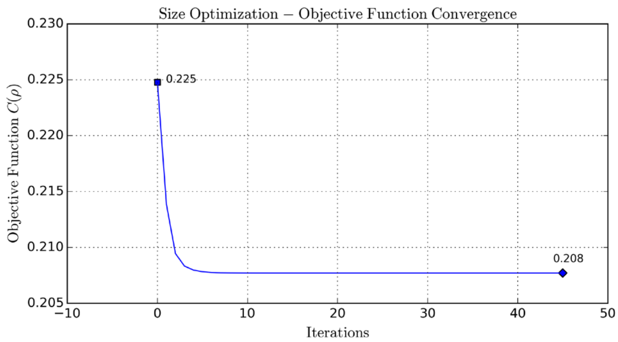

Figure 9: Objective function convergence during size optimization

One can conduct sensitivity analysis to find optimal volume fraction and discretization for the problem. However, the goal of this paper is to provide an approach to generating CAD models, less emphasis is placed on achieving very precise topology-optimized solutions.

The 2D binary image data obtained from the topology optimization is skeletonized and the frame model is extracted as discussed in sections 2.2 and 2.3. Fixed-ended pixels and force-applied pixels were tagged to as not to alter them during any processes. A merge ratio of 0.1 is used to remove short members and a constrain-angle of 10° is used to align the members with less than a 10° angle.

Size optimization is done for the extracted frame model by taking the thickness of the plate as 4 cm. Table 3 gives the parameters used for the size optimization and Figure 9 shows the convergence of the objective function.

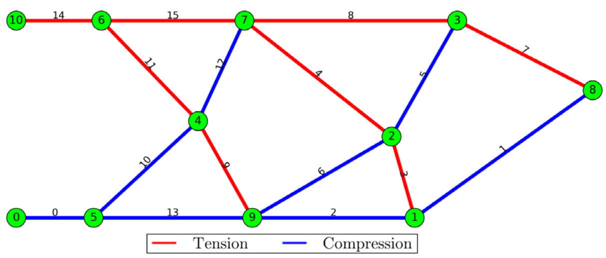

Figure 10: Axial stress types of members

Table 4: Size optimization results

Member	A_{opt} cm^2	I_{opt} cm^4	Category	Alternate SHS Section
0	31.48	78.86	Bottom	SHS 140
1	23.07	42.35		SHS 100
2	25.82	53.05		SHS 120
13	26.03	53.92		SHS 120
7	26.38	55.38	Top	SHS 120
8	23.34	43.35		SHS 120
14	30.84	75.69		SHS 140
15	26.9	57.58		SHS 120
3	30.89	75.93	Web	SHS 140
4	24.54	47.92		SHS 120
5	27.48	60.09		SHS 120
6	25.88	53.3		SHS 120
9	28.94	66.65		SHS 140
10	26.95	57.8		SHS 120
11	28.95	66.69		SHS 140
12	27.11	58.49		SHS 120

Lower and upper bounds of the area are taken as circular areas with a radius of 1 cm and 4 cm, respectively. The initial sizes of the members are assumed to have the same cross-sectional area of $A = 26.46$ cm^2 which is found by diving the total volume $V_{(A)} = 0.5V = 15600$ cm^3 by the total length of the members. Using these parameters, the sizes of the members were optimized and Table 4 gives the obtained results of optimization. Figure 11 shows the axial stress types (tension or compression) of the members.

Moreover, the possibility of using Square Hollow Sections (SHS) of thickness 6.3 mm as alternatives was analyzed and more suitable sections were suggested based on the size optimization results. Similarly, one can analyze various sections and the model can be structurally designed based on standard codes of practice. The authors are presently working on this implementation to automate the process.

Finally, a CAD model of the structures can be generated for manufacturing with obtained node, member and cross-section details of the final optimized structure.

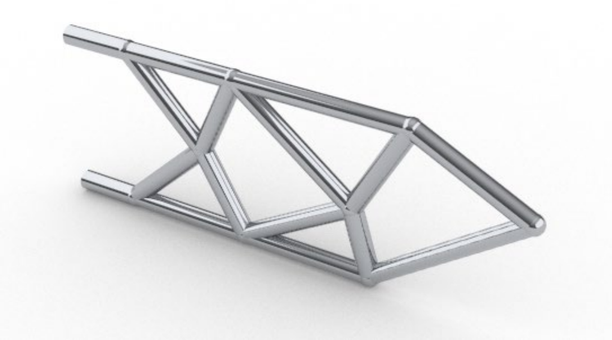

Figure 11: Rendered view of the CAD model

4. Conclusion

Currently, there are only a few unified frameworks that generate ready-to-manufacture parametric CAD models of structurally optimum designs. In this paper, a novel approach to obtaining structurally optimized outputs of Civil engineering structures were presented. The two-dimensional structural problem is the first topology optimized and the solid-void 2D binary image result was obtained. Obtained binary data is converted to 1-pixel wide chain using a skeletonization algorithm and converted to a graph model. Then, The graph model is modified to obtain a structurally feasible frame model and the sizes of the members were optimized. Finally, for a chosen section, a manufacturable CAD model is generated by recursively combining primitive shapes using boolean operations.

This workflow is entirely developed in a single parametric platform called Rhino-Grasshopper. The authors presently working on nodal-based layout optimization to obtain more optimized models and design verification methods to assess the structural performance of structures according to standard codes of practice.

Acknowledgement

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